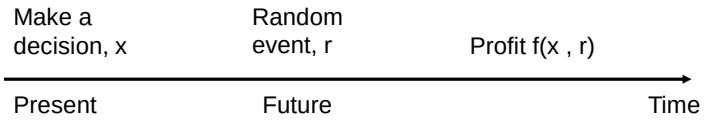
Risk aversion

Decision making under uncertainty



- The decision maker often has no control over the random event
  - Financial Risks: stock prices, interest rates, currency rates, commodity prices, defaults, etc.
  - Market Risks: demand, input costs, prices
  - Political Risks: strikes, privatization
  - Natural Risks: weather, natural disasters
  - Lottery: Pick the right colour and win lunch money

Expected value

- Setup

  x_i uncertain outcome of a gamble
  p_i probability of the corresponding uncertain outcome

- Expected Value

$$Expected\ Value = \sum Outcome \cdot Probability$$

$$E(x) = \sum x_i \cdot p_i$$

Expected values do not reflect the risk in decision

- Firms' and individuals' choices often deviate from Expected Value
  - Buying insurance, lotteries
  - Diversifying assets, activities
- Why?
  - Cash constraints
  - Value is not linear with payoff
  - Large losses may hurt too much
  - Relative preference for certainty/distaste for uncertainty

Expected value (EV)

- Setup

  x_i uncertain outcome of a gamble
  p_i probability of the corresponding uncertain outcome

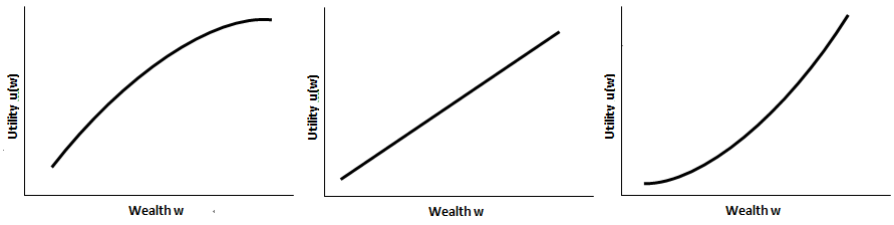
- Expected Value

$$Expected\ Value = \sum Outcome \cdot Probability$$

$$E(x) = \sum x_i \cdot p_i$$

Risk attitudes

Risk Averse | Risk Neutral | Risk Seeking



Expected utility (EU)

- Setup

x_i	uncertain outcome of a gamble
p_i	probability of the corresponding uncertain outcome
w_0	initial wealth
$w_i = x_i + w_0$	wealth for the corresponding uncertain outcome

- Expected utility
  - If your preferences over gambles satisfy a few very reasonable axioms of coherence (ordering, transitivity, continuity, independence, substitutability, and monotonicity), then:
  - There exists utility function over wealth $u(w_i)$ such that you should make decisions to maximize

$$Expected\ Value = \sum Utility\ of\ Wealth \cdot Probability$$

$$Eu(w) = \sum u(w_i) \cdot p_i$$

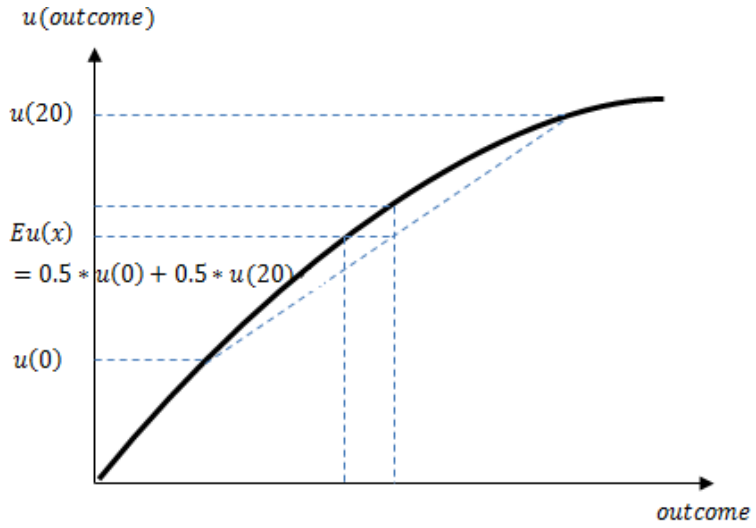
Decisions under expected utility

- **Gamble**: $(p_1, w_1; p_2, w_2; \ldots; p_n, w_n)$.
- **Expected utility of gamble**: $Eu(w) = \sum p_i u(w_i)$.
  - It measures the worth of the gamble in *utility units,* and reflects the *risk* in payoffs distribution.
  - Caution: $EU \neq u(EV)$, i.e., $\sum p_i u(w_i) \neq u(\sum p_i w_i)$, unless $u(\cdot)$ is linear!
- **Decision rule**: choose the gamble that yields the largest expected utility $Eu(w)$.

Certainty equivalent (CE)

- Definition
  - Certainty Equivalent is a sure payoff worth as much as the gamble,
    $$u(CE + w_0) = Eu(w)$$
  - CE provides a risk-adjusted measure of a gambles' value, expressed in the *same units* as the payoffs.
- Interpretation
  - You are *indifferent* between receiving CE for sure or taking the gamble.
- **Risk premium (RP):** the minimum amount of money by which the expected outcome of a gamble must exceed the known fixed amount of money, in order to induce an individual to choose the uncertain outcome over the fixed amount.
  $$RP = E(x) - CE$$

- Assuming initial wealth $w_0 = 0$



Finance and risk

- The “certainty equivalent” is a government bond
- Pays agreed upon amount of interest
- Minimal risk
- Risky projects have to pay a risk premium to their investors to make them invest (instead of investing into the government bond)

→ This determines how much profit your investors expect to make

Adding time

- The safe government bond pays interest i
  Investment I into the bond becomes $I * (1 + i)$ after 1 year
  $I * (1 + i)^t$ after t years
- Would you invest I into a project that pays you x after t years?

- What if the investment is risky?